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Laplace Transforms Method to Solve Differential Equation

A Graduation project submitted to the Mathematics department in partial of the requirements for the degree bachelors in Mathematics fulfillment

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بسم الله الرحمن الرحيم

﴿أَلَا إِنَّ أَوْلِيَاءَ اللَّهِ لَا خَوْفٌ عَلَيْهِمْ وَلَا هُمْ يَحْزَنُونَ﴾

[يونس:٦٢]

صدق الله العظيم

الإهداء

إلى الوالدين .... فلولاهما لما وجدت في هذه الحياة، ومهما تعلمت الصمود، مهما

كانت الصعوبات

إهداء

إلى أساتذتي الكرام......... فعنهم استقيت الحروف وتعلمت كيف أنطق الكلمات، وأصوغ العبارات واحتكم إلى القواعد في مجال.....

إهداء

إلى الزملاء والزميلات الذين لم يدخروا جهدا في مدي بالمعلومات والبيانات

أهدي إليكم بحتى هذا

داعيا المولى سبحانه وتعالى أن يتكلل بالنجاح والقبول من جانب أعضاء

لجنة المنافسة المبجلين

الشكر والتقدير

لحمد لله رب العالمين والصلاة والسلام على سيد الأولين والآخرين وأشرف الخلق

أجمعين محمد وعلى آله وصحبه وسلم تسليما كثيراً

اما بعد.......

يطيب لي أن أتقدم بجزيل الشكر والثناء إلى من لا أجد كلمة في سطور الكتب تستحق شرف الارتقاء لشكره ، إلى أستاذتي المشرفة (م . م ايناس حسن عبد) التي كانت نعم العون لي ، لما أبدته من توجيهات وملاحظات علمية وما منحتني إياه من وقت وجهد نورت طريق بحثي العلمي.

ومن واجب الاخلاص والعرفان أن أتقدم بالشكر والامتنان إلى الأستاذي الأفاضل (دكتور خالد هادي حميد)، لما قدمه لي من توجيهات وأراء سديدة خلال مدة الدراسة والبحث.

وأتوجه بالشكر الى كل من تكرم وسمح بتطبيق الدراسة عليه، و لما قدموه لي من خدمات جليلة لن أنساها

وأخيراً أتقدم بالشكر الى كل من شارك بمساعدة، أو مشورة ، أو رأي، أو ملاحظة.

والله ولي التوفيق

CONTENTS

INTRODICTION:

Since the time of Newton, differential equations are still used in the understanding of the physical, engineering, and biological sciences, as well as their contribution to the study of mathematical analysis. Hence, it can be said without going beyond or exaggerating that the differential equations extend their influence to include many medical and social sciences such as psychology, economics and sociology, as most of the relationships and laws governing the variables of any engineering or physical issue appear in the form of differential equations. To understand these problems it was necessary to solve these differential equations, The Laplace transform is one of the ways to solve these equations The Laplace transform is a process that takes place on the mathematical functions to convert them from one field to another, usually the conversion from the time domain to the frequency domain, which is similar to the Fourier transform, but it is developed independently. The Laplace transform is useful in analyzing linear systems (unlike the Fourier transform, which is usually used in signal analysis), and it is also used to solve differential equations because it transforms them into algebraic equations. The transformation is called by this name in relation to the French scientist Pierre Laplace, who lived in the nineteenth century, who was the first to "study the properties of the Laplace equation, which takes the following form[1] $\nabla^2 \psi = 0$

Where ∇^2 An effective Laplace symbol for any scalar mathematical function When solving many applied engineering problems, some types of boundary or initial .value problems sometimes appear For example, some differential equations appear in which the non-homogeneous term, G(x) is in the form of impulses, or | In the form of continuous functions (Pieswise - Continuous), which cannot be used with traditional methods to solve these types of problems. Thus we have to look for other mathematical methods and methods to deal with such equations. One of these methods uses what is known as the "Laplace transform After the French mathematician Pierre-Simon Laplace (1827 - 1749,Laplace, P. c).

Transformation in general is a tool (Devise) for converting functions and equations from their original form to another simpler form, or at least to another form that is known to us. These and transformations are usually integral transformations, such as Laplace transforms, Fourier transforms, Laguerre transforms, and many others. The Laplace transform is an integral transformation that when it affects the function turns it into another function completely different from the original function As the independent variable of the original function is converted to another variable, the scope and extent of the original function change. Thus, he transforms marginal or elementary problems into algebraic equations, and the transformations are usually integrative transformations, .such as Laplace transforms, Fourier transforms, and Laguerre transforms And many more. The Laplace transform is an integral transformation that when it affects the function turns it into another function completely different from the original function, where the independent variable of the original function is converted to another | variable, and thus the scope and extent of the original function changes. Thus, we can convert the function we are dealing with from its complex form to another form, perhaps simpler and easier to deal with than the original function. For example, the function $f(t) = \cos(at)$ Where the range is R by the effect of the Laplace transform on it .".is transformed into the rational function [2]

$$F(S) = \frac{S}{S^2 + a^2}$$

However, the greatest benefit of the Laplace transform lies in its ability to solve marginal or elementary problems associated with any type of differential equation, where the effect of the Laplace transform on the differential equation can be transformed into an algebraic equation in which the unknown is the Laplace transform while he is holding the solution of the equation under his grip and influence, and by solving This algebraic equation can get an explicit Laplace transform. Then by finding | The inverse Laplace transform We can free the solution from the grip of the Laplace The equation transform and get the solution of the original differential equation

Research problem

Sometimes we find that there are some problems in differential equations that are difficult to solve by known methods, so we resort to other ways to solve these .equations, including solving the differential equation using the Laplace transform [3]

Research importance

Laplace transform helps to solve continuous functions at intervals whose solutions can | be obtained using traditional methods

When changing the form of the original complex function to another form that is easier and simpler to deal with by converting the differential equation into an algebraic equation that can be solved, and by finding the inverse Laplace transform, we get the | .solution of the original differential equation[2]

.research aims

1- Learn about Laplace and Laplace Transforms

2-Solving differential equations that are difficult to solve by ordinary methods

3- Laplace transform in solving initial value problems given by linear differential equations with constant coefficients [4]

Research Methodology

. In this research, researchers use the descriptive method and the experimental method[4]

Chapter One

Laplace Transform

Chapter One Laplace Transform

1.1.LAPLACE TRANSFORM

The Laplace transform can be used to solve differential equations. Besides being a different and efficient alternative to variation of parameters and undetermined coefficients, the Laplace method is particularly advantageous for input terms that are piecewise-defined, periodic or impulsive. The direct Laplace transform or the Laplace integral of a function f(t) defined for $0 \leq t < \infty$is the ordinary calculus integration problem

$$\int_0^\infty f(t)e^{-st}dt \quad \dots\dots\dots.(1.1)$$

succinctly denoted $\mathcal{L}$(f(t)) in science and engineering literature. The $\mathcal{L}$ -notation recognizes that integration always proceeds over $t = 0 \;\; to$

$t = \infty$ and that the integral involves an integrator e^{-st}dt instead of the usual dt. These minor differences | distinguish Laplace integrals from the ordinary integrals found on the inside covers of calculus texts. [6]

The foundation of Laplace theory is Lerch's cancellation

$$\int_0^\infty y(t)e^{-st}dt = \int_0^\infty f(t)e^{-st} \text{ implies } y(t) = f(t) \quad \dots\dots\dots.(1.2)$$

Or

$$\mathcal{L}\,(y(t)= \mathcal{L}(f(t)) \;\; implies \;\; y(t) = f(t)$$

In differential equation applications, y(t) is the sought-after unknown while f(t) is an explicit expression taken from integral tables. Below, we illustrate Laplace's method by solving the initial value problem

y' = -1, y(0) = 0 ……… (1.3)

The method obtains a relation $\mathcal{L}$ (y(t)) = ($\mathcal{L}$ -t), whence Lerch's cancellation law implies the solution is y(t) = -t.

Chapter tow Laplace method 2.1 is advertised as a table lookup method, in which the solution y(t) to a differential equation is found by looking up the answer in a special integral table.

Laplace method L -notation details for y' = -1, y(0) = 0 translated from

L(y' (t)) = L(-1) Apply L across y' = -1, or multiply y' = - 1 by e^{-st}, integrate t = 0 to t = ∞ L(y' (t)) = -1/s

S $\mathcal{L}$ (y(t)) – y(0) = -1/s Integrate by parts on the left .

$\mathcal{L}$ (y(t)) = -1/s^2 Use y(0) = 0 and divide.

$\mathcal{L}$(y(t)) = L(-t) Apply Table 1.

y(t) = -t Invoke Lerch's cancellation law

Example..1.1 (Laplace method) Solve by Laplace's method the initial value problem

$$y' = 5 - 2t, \; y(0) = 1 \ldots\ldots (1.4)$$

Solution:

Laplace's method is outlined in . The $\mathcal{L}$ -notation of will be used to find the solution

$y(t) = 1 + 5t - t^2$.

$\mathcal{L}(y'(t)) = \mathcal{L}(5 - 2t)$ Apply $\mathcal{L}$ across $y' = 5 - 2t$

$[(y'(t)) = \frac{5}{s} - \frac{2}{s^2}$ Use Table 1.

$S\,\mathcal{L}(y(t)) - y(0) == \frac{5}{s} - \frac{2}{s^2}$ Apply the t -derivative rule .

$\mathcal{L}(y(t)) = = \frac{1}{s} + \frac{5}{s^2} - \frac{2}{s^3}$ Use y(0) = 1 and divide.

$\mathcal{L}(y(t)) = \mathcal{L}(1) + 5\,\mathcal{L}(t) - L\,\mathcal{L}(t2)$ Apply Table 1 , backwards.

$= \mathcal{L}(1 + 5t - t2)$ Linearity .

$y(t) = 1 + 5t - t^2$ Invoke Lerch's cancellation law

Example.1.2

(Laplace method) Solve by Laplace's method the initial value problem

$$y'' = 10, y(0) = y'(0) = 0 \ldots\ldots\ldots (1.5)$$

Solution:

The $\mathcal{L}$ -notation of will be used to find the solution

y(t) = $5t^2$. $\mathcal{L}$ (y"(t)) = $\mathcal{L}$ (10) Apply $\mathcal{L}$ across y' = 10

S $\mathcal{L}$ (y'(t)) – y'(0) = L(10) Apply the t-derivative rule to y', that is, replace y by y'.

s[s $\mathcal{L}$ (y(t)) – y(0)] – y'(0) = $\mathcal{L}$ (10) Repeat the t -derivative rule, on y.

s^2 $\mathcal{L}$ (y(t)) = $\mathcal{L}$ (10) Use y(0) = y'(0) = 0.

$\mathcal{L}$ (y(t)) =$\frac{10}{s^3}$. Use Table 1. Then divide.

$\mathcal{L}$ (y(t)) = $\mathcal{L}$ ($5t^2$). Apply Table 1 , backwards

y(t) = $5t^2$ Invoke Lerch's cancellation law.

1.2.Laplace Integral.

The integral $\int_0^\infty g(t)e^{-st}\,dt$ is called the Laplace integral of the function $g(t)$. It is defined by$\lim_{N\to\infty}\int_0^N g(t)e^{-st}\,dt$ and depends on variable s. The ideas will be illustrated for $g(t)$= 1, $g(t)$= t and $g(t)$= t^2, producing the integral formulas in Table 1 .

$\int_0^\infty (1)e^{-st}$ = -(1/s)e^{-st} $I_{t=0}^{t=\infty}$ Laplace integral of $g(t)$= 1.

= 1/s Assumed s > 0

$\int_0^\infty (t)e^{-st}\,dt = \int_0^\infty -\frac{d}{ds}(e)^{-st}\,dt$ Laplace integral of $g(t)$ = t.

=$-\frac{d}{ds}\int_0^\infty (1)e^{-st}$ dt Use $\int \frac{d}{ds} f(t,s)\,dt$ =$\frac{d}{ds}\int f(t,s)\,dt$

$= -\frac{d}{ds}$ (1/s) Use $\mathcal{L}$ (1) = 1/s

= $1/s^2$ Differentiate .

$\int_0^\infty (t^2)e^{-st}\,dt = \int_0^\infty -\frac{d}{ds}(te^{-st})$ dt Laplace integral of $g(t)$ = t^2

$= -\frac{d}{ds}\int_0^\infty (t)e^{-st}\,dt$

= - ($1/s^2$) Use $\mathcal{L}$ (t) = $1/s^2$

= $2/s^3$

Remark.1.

The Laplace integral $\int_0^\infty g(t)e^{-st}$dt for $g(t)$= 1, t and t^2.

$\int_0^\infty (t)e^{-st}$dt = $\frac{1}{s}\int_0^\infty (t)e^{-st}$dt = $\frac{1}{s^2}\int_0^\infty (t^2)e^{-st}\,dt = \frac{2}{s^3}$ In summary

$\mathcal{L}$ (t^n) = $\frac{n!}{s^{1+n}}$

An Illustration. The ideas of the Laplace method will be illustrated for the solution y(t) = -t of the problem y' = -1, y(0) = 0. The method, entirely different from variation of parameters or undetermined coefficients, uses basic calculus and college algebra.

Remark.2.

Laplace method details for the illustration

y' = -1, y(0) = 0.

$y'(t)e^{-st} = -e^{-st}$ Multiply y' = -1 by e^{-st}.

$\int_0^\infty y'(t)e^{-st}$dt=$\int_0^\infty -e^{-st}\,dt$ Integrate t = 0 to t = ∞.

$\int_0^\infty y'(t)e^{-st}$**dt = -1/s Use Table 1**

$s\int_0^\infty y(t)e^{-st}$**dt – y(0) = -1/s Integrate by parts on the left .**

$\int_0^\infty y(t)\, e^{-st}$**dt = -1/$s^2$ Use y(0) = 0 and divide.**

$\int_0^\infty y(t)\, e^{-st}dt$ **=** $\int_0^\infty(-t)e^{-st}$**dt Use Table 1.**

$y(t) = -t$ Apply Lerch's cancellation law.

Existence of the Transform. The Laplace integral $\int_0^\infty e^{-st}f(t)$dt is known to exist in the sense of the improper integral definition

$$\int_0^\infty g(t)dt = \lim_{N\to\infty}\int_0^N g(t)\,\text{dt} \quad ......(1.6)$$

provided f(t) belongs to a class of functions known in the literature as functions of exponential order. For this class of functions the relation [8]

$$(2)\quad \lim_{t\to\infty}\frac{f(t)}{e^{at}} = 0$$

is required to hold for some real number a, or equivalently, for some constants M and a,

(3) [f(t) ≤ Me^{at}

In addition, f (t) is required to be piecewise continuous on each finite subinterval of$0 \le t < \infty$, a term defined as follows.

Definition 1.1

(piecewise continuous) A function $f(t)$is piecewise continuous on a finite interval [a, b] provided there exists a partition $a = t0 < \ldots <$ tn = b of the interval [a, b] and functions $f1, f2, \ldots, fn$ continuous on $(-\infty, \infty)$ such that for t not a partition point

$$f(t) = \begin{cases} f_1(t) & t_0 & < t < t_1 \\ . & & . \\ . & & . \\ . & & . \\ f_{(n)}(t) & t_{n-1} & < t < t_n \end{cases}$$

Example 1.3

(Exponential order) Show that $f(t)= e^t \ cos \ t + t$ is of exponential order, that is, show that $f(t)$ is piecewise continuous and find α > 0 such that $\lim_{t\to\infty} f(t)/e^{\alpha t}$ = 0

Solution: Already, $f(t)$is continuous, hence piecewise continuous. From L'Hospital's rule in calculus, $\lim_{t\to\infty} p(t)/e^{\alpha t}$= 0 for any polynomial p and any α > 0. Choose α = 2, then

$$\lim_{t\to\infty} \frac{f(t)}{e^{2t}} = \lim_{t\to\infty} \frac{\cos t}{e^t} + \lim_{t\to\infty} \frac{t}{e^{2t}} = 0$$

Theorem 1.1

Let f(t) be piecewise continuous on every finite interval in t > 0 and satisfy $f(t)$< $Me^{\alpha t}$ for some constants M and α. Then $\mathcal{L}$ (f (t)) exists for

s $>$ α and $\lim_{s\to\infty} \boldsymbol{\mathcal{L}(f(t)) = 0}$

Proof: It has to be shown that the Laplace integral of f is finite for $s > \alpha$ Advanced calculus implies that it is sufficient to show that the integrand is absolutely bounded above by an integrable function

$$\int_0^\infty g(t)\, dt = \frac{M}{s-\alpha}$$

Inequality $|f(t)| \leq Me^{\alpha t}$ implies the absolute value of the Laplace transform integrand $f(t)e^{-st}$ is estimated by

$$If(t)e^{-st} | \leq Me^{\alpha t}e^{-st} = g(t).$$

The limit statement follows from $\left|\boldsymbol{\mathcal{L}}\,(\boldsymbol{f}(\boldsymbol{t}))\right| \leq \int_0^\infty g(t)dt = \frac{M}{s-\alpha}$, because the right side of this inequality has limit zero at $s = \infty$. The proof is complete.[9]

Theorem 1.2

(Lerch) If $f_1(t)$ *and* $f_2(t)$ are continuous, of exponential order and $\int_0^\infty f_1(t)e^{-st}\ dt = \int_0^\infty f_2(t)e^{-st}\ dt$ for all $s > s_0$, then = $f_1(t) = f_2(t)$. For $t \geq 0$.

Theorem 1.3

(t-Derivative Rule) If $f(t)$ is continuous $\lim_{t\to\infty} f(t)e^{-st} = 0$ for all large values of s and $f'(t)$ is piecewise continuous, then $\mathcal{L}(f'(t))$ exists for all large s and $\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)$

1.3.Laplace Integral

$$\int_0^\infty (t^n)e^{-st}\,dt = \frac{n!}{s^{1+n}} \qquad \mathcal{L}(t^n) = \frac{n!}{s^{1+n}}$$

$$\int_0^\infty (e^{an})e^{-st}\,dt = \frac{1}{s-a} \qquad \mathcal{L}(e^{at}) = \frac{1}{s-a}$$

$$\int_0^\infty (\cos bt)e^{-st}\,dt = \frac{s}{s^2+b^2} \qquad \mathcal{L}(\cos bt) = \frac{s}{s^2+b^2}$$

$$\int_0^\infty (\sin bt)e^{-st}\,dt = \frac{b}{s^2+b^2} \qquad \mathcal{L}(\sin bt) = \frac{b}{s^2+b^2}$$

$\mathcal{L}\big(H(t-a)\big) = \frac{e^{-as}}{s}$ $(a \geq 0)$ *Heaviside unit step, defined*

by $$H(t) = \begin{cases} 1 & for \quad t \geq 0 \\ 0 & otherwise \end{cases}$$

$\mathcal{L}(\delta(t-a)) = e^{-at}$ Dirac delt a $\delta(t) = dh(t)$ special usage rules apply

$\mathcal{L}(floor\left(t/a\right)) = \frac{e^{-as}}{s(1-e^{-as})}$ *Staircase fun c t ion ,*

$floor(x) = greatest\ integer\ \leq x.$

$\mathcal{L}(sqw(t/a)) = \frac{1}{s}\tanh(as/2)$ *Square wave ,*

$sqw(x) = (-1)^{floor(x)}.$

$\mathcal{L}(atrw(t/a)) = \frac{1}{s^2}tanh(as/2)$ *Triangular wave,*

$trw(x) = \int_0^x sqw(r)dr.$

$\mathcal{L}(t^a) = \frac{\Gamma(1+a)}{s^{1+a}}$ *Generalized power function,*

$\Gamma(1+a) = \int_0^\infty e^{-x}x^a\,dx$

$\mathcal{L}\left(t^{-1/2}\right) = \sqrt{\frac{\pi}{s}}$ *Because* $\Gamma(1/2) = \sqrt{\pi}$

Examples 1.4.1

Let $f(t) = t(t-1) - \sin^{2t + e^{3t}}$. *Compute* $\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}(\boldsymbol{t})\right)$ *using*

the basic Laplace table and transform linearity properties

Solution :

$$\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}(\boldsymbol{t})\right) = \boldsymbol{\mathcal{L}}\,(\boldsymbol{t}^2 - \mathbf{5}\boldsymbol{t} - \sin 2\boldsymbol{t} + \boldsymbol{e}^{3\boldsymbol{t}})\ \textbf{\textit{Expand}}\ \boldsymbol{t}(\boldsymbol{t} - \mathbf{5})$$

$$= \left(\boldsymbol{\mathcal{L}}\boldsymbol{t}^2 - \boldsymbol{\mathcal{L}}5\boldsymbol{t} - \boldsymbol{\mathcal{L}}(\boldsymbol{sin}\ 2\boldsymbol{t}) + \boldsymbol{\mathcal{L}}(\boldsymbol{e}^{3\boldsymbol{t}}\right)\ \textbf{\textit{Linearity applied}}.$$

$$= \frac{\mathbf{2}}{\boldsymbol{s}^3} - \frac{\mathbf{5}}{\boldsymbol{s}^2} - \frac{\mathbf{2}}{\boldsymbol{s}^2 + \mathbf{4}} - \frac{\mathbf{1}}{\boldsymbol{s} - \mathbf{3}}\ \textbf{\textit{Table lookup}}.$$

***Example*1.4.2**

(*Inverse Laplace transform*) *Use the basic Laplace table backwards plus transform linearity properties to solve for* $f(t)$ *in the equation*

$$\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}(\boldsymbol{t})\right) = \frac{\boldsymbol{s}}{\boldsymbol{s}^2 + \mathbf{16}} + \frac{\mathbf{2}}{\boldsymbol{s} - \mathbf{3}} + \frac{\boldsymbol{s} + \mathbf{1}}{\boldsymbol{s}^3}$$

Solution:

$$\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}(\boldsymbol{t})\right) = \frac{\boldsymbol{s}}{\boldsymbol{s}^2 + \mathbf{16}} + \mathbf{2}\frac{\mathbf{2}}{\boldsymbol{s} - \mathbf{3}} + \frac{\mathbf{1}}{\boldsymbol{s}^2} + \frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{2}}{\boldsymbol{s}^3}$$

Convert to tabie entries

$$\boldsymbol{\mathcal{L}}\,(\cos 4\boldsymbol{t}) + 2\boldsymbol{\mathcal{L}}(\boldsymbol{e}^{3\boldsymbol{t}}) + \frac{1}{2}\ \boldsymbol{\mathcal{L}}(\boldsymbol{t}^2)$$

Laplace table (***backwards***)

$$= \mathcal{L}\left(\cos 4t + 2e^{3t} + t + \frac{1}{2}t^2\right)$$

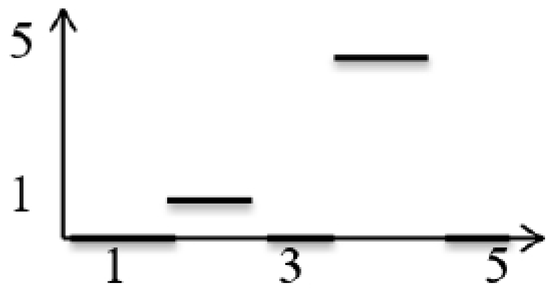
Linearity applied

$$f(t) = \cos 4t + 2e^{3t} + t + \frac{1}{2}t^2$$

Lerch ś Cancellation law

Example 1.4.3

(Heaviside) find the Laplace transform of $f(t)$ in figure 1.1



$$\boldsymbol{figure\ 1.1\ A\ \ piecewise\ defined\ function}$$

$$\boldsymbol{f(t) on\ 0 \ \le t < \infty : f(t) = 0\ except\ for\ 1 \le t < 2\ and}$$

$$\boldsymbol{3 \le t < 4}$$

Solution :

The details require The use of the Heaviside function formula

$$H(t-a)-H(t-b)=\begin{cases}1 & a\le t<b\\ 0 & otherwise\end{cases}$$

$$The\ \ formula\ for\ f(t):$$

$$f(t)=\begin{cases}1 & 1\le t<2\\ 5 & 3\le t<4\\ 0 & otherwise\end{cases}=\begin{cases}1 & 1\le t<2\\ 0 & otherwise\end{cases}+5\begin{cases}1 & 3\le t<4\\ 0 & otherwise\end{cases}$$

Then $f(\mathrm{t})=f_1(t)+5f_2(t)\ where$

$f_1(t)=H(t-1)-H(t-2)\qquad and$

$f_2(t)=H(t-3)-H(t-4)\,.The\ extended\ table\ gives$

$$\mathcal{L}\big(f(t)\big)=\mathcal{L}(f_1(t))+5\mathcal{L}\,(f_2(t))\qquad Linearity.$$

$$=\mathcal{L}(H(t-1))-\mathcal{L}(H(t-2))+5\mathcal{L}\,(f_2(t))\qquad substitute$$

$$for\ f_1=\frac{e^{-s}-e^{-2s}}{s}+5\mathcal{L}\,(f_2(t))\ \ Extended\ table\ user$$

$$=\frac{e^{-s}-e^{-2s}+5e^{-3s}-5e^{-4s}}{s}\qquad Similarly\ for\ f_2\,.$$

***Example* 1.4.4**

A machine shop tool that repeatedly hammers a die is modeled by the Dirac impulse model $\boldsymbol{f(t)=\sum_{n=1}^{N}\delta(t-n)}$.

show that $\mathcal{L}(f(t))=\sum_{n=1}^{N}e^{-ns}$.

Solution:

$$\boldsymbol{\mathcal{L}(f(t))=\mathcal{L}\left(\sum_{n=1}^{N}\delta(t-n)\right)}$$

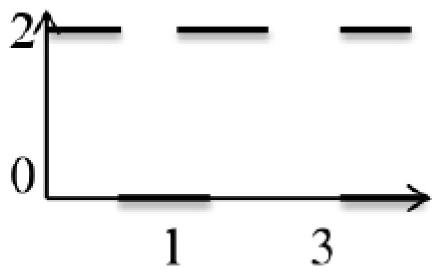
$$=\sum_{n=1}^{N}\mathcal{L}(\delta(t-n)) \qquad \textbf{\textit{Linearity}}$$

$$=\sum_{n=1}^{N}e^{-ns} \qquad \textbf{\textit{Extended Laplace table}}$$

***Example* 1.4.5**

(Square wave) Aperiodic camshaft force $f(t)$ *applied to a mechanical system has the idealized graph shown in figure* 1.2. *show that* $f(t)=1+sqw(t)$ *and*

$$\mathcal{L}(f(t))=\frac{1}{s}\,(1+\tanh(s/2))$$



$$\boldsymbol{Figure\ 1.2\ Aperiodic\ \ force\ \ f(t)\ applied\ to\ \ a\ mechanical\ system}$$

Solution:

$$1 + sqw(t) = \begin{cases} 1 + 1 2n \le t < 2n + 1 & , n = 0,1,2 \dots \dots \\ 1 - 1 2n + 1 \le t < 2n + 2 & , n = 0,1,2, \dots .. \end{cases}$$

$$= \begin{cases} 2 & 2n \le t < 2n + 1 \quad , n = 0,1,2, \dots . \\ 0 & otherwise \end{cases}$$

$$= f(t).$$

By the extended Laplace tabe

$$\mathcal{L}\left(f(t)\right) = \mathcal{L}(1) + \mathcal{L}\big(sqw(t)\big) = \frac{1}{s} + \frac{\tanh({}^{s}/_{2})}{s} .$$

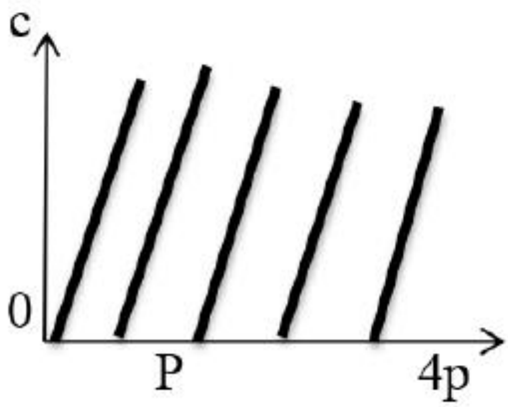
***Example* 1. 4. 6**

$(sawtooth\ wave\ )\ Express\ the\ P-periodic\ sawtooth$

$wave\ \ represented\ in\ \text{figure}1.3\ as$

$f(t) = ct/P - c\ \text{floor}\ (t/P)\ \ and\ obtain\ the\ formula$

$$\mathcal{L}\left(f(t)\right) = \frac{c}{ps^2} - \frac{ce^{-Ps}}{s - se^{-Ps}}$$



$figure\ 1.3\ AP - periodic\ sawtooth\ wave$

$f(t)\ \ of\ height\ \ c > 0$

Solution:

The representation originates from geometry, because the periodic function f can be viewed as derived from ct/P by subtracting the correct constant from each of intervals

[P, 2P], [2P, 3P], etc. The technique used to verify the identity is to define $g(t)$ *= ct/P – c floor (t/P) and then show that g is P -periodic and* $f(t) = g(t)$ *on* 0 *≤t< P. Two P periodic functions equal on the base interval 0 ≤ t < P have to be identical, hence the representation follows. The fine details: for 0 ≤ t< P, floor* $(t/P) = 0$ *and floor (t/P+ k) = k. Hence* $g(t + kP) = ct/P + ck$ *–* c *floor (k) = ct/P =* $g(t)$*, which implies that g is P -periodic and g (t) = f(t) for 0 ≤t<P.*

$$\mathcal{L}(f(t)) = \frac{c}{p}\mathcal{L}(t) - c\mathcal{L}\left(floor\left({}^{t}/_{p}\right)\right) \qquad \textbf{linearity}.$$

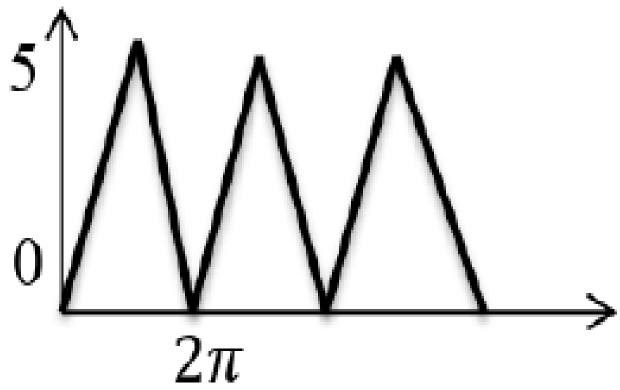
$$= \frac{c}{ps^2} - \frac{ce^{-ps}}{s - se^{-ps}}$$

Basic and extended table applied.

Example 1.4.7

(Triangular wave) Express the triangular wave f of Figure 4 in terms of the square wave sqw and obtain

$$\mathcal{L}(f(t)) = \frac{5}{\pi s^2}\tanh(\pi s/2).$$



$Figure\ 1.4.$

$A\ 2\pi\ -periodic\ triangular\ wave\ f(t) of\ height\ 5.$

Solution:

The representation of f in terms of sqw is

$f(t) = 5\int_0^{t/\pi} sqw(x)dx.$

Details: A 2-periodic triangular wave of height 1 is obtained by integrating the square wave of period 2. A wave of height c and period 2 is given by

$$c\ trw\ (t) = C\int_0^t sqw(x)dx.$$

$$Then\ f(t) = ctrw\left(\frac{2t}{P}\right) = C\int_0^{2t/P} sqw(x)dx$$

$$where\ c\ = 5\ and\ P\ = 2\pi$$

$$Laplace\ transform\ details:$$

$$Use\ the\ extended\ Laplace\ table\ as\ follows.$$

$$\boldsymbol{\mathcal{L}}\big(f(t)\big) = \frac{5}{\pi}\boldsymbol{\mathcal{L}}\left(\pi\ trw\left(\frac{t}{\pi}\right)\right) = \frac{5}{\pi s^2}\ tanh\left(\frac{\pi s}{2}\right).$$

Chapter two

Laplace Transform Rules

Chapter two Laplace Transform Rules

2.1 Laplace Transform Rules

$$\mathcal{L}(f(t)+g(t))=\mathcal{L}(f(t)+\mathcal{L}(g(t)) \qquad Linearity.$$

$The\ Laplace\ of\ a\ sum\ of\ the\ Laplaces\ .$

$$\mathcal{L}(cf(t))=c\mathcal{L}(f(t)) \qquad Linearity.$$

$Constans\ move\ through\ the\ \ \mathcal{L}--symbol.\ .$

$$\mathcal{L}\left(y'(t)\right)=s\,\mathcal{L}(y(t))-y(0)\ \ The\ t-derivative\ rule\ .$$

$derivatives\ \ \mathcal{L}\,(y')\ \ are\ replaced\ \ln\ transfomed\ \ \ equations.$

$$\mathcal{L}\left(\int_0^t g(x)\,dx\right)=\frac{1}{s}\mathcal{L}\left(g(t)\right) \qquad The-integral\ \ rule\ .$$

$$\mathcal{L}\left(tf(t)\right)=-\frac{d}{ds}\mathcal{L}\left(f(t)\right) \qquad The\ \ s\ \ differentiation\ rule\ .$$

$Multiplylng\ f\ by\ t\ applies\ -\frac{d}{ds}\ \ to\ the\ transform\ of\ f$

$first\ shifting\ rule.$

$$\mathcal{L}\left(e^{at}f(t)\right)=\mathcal{L}(f(t))\lfloor s\to(s-a) \qquad Multiplying\ \ f$$

$by\ e^{at}\ replaces\ \ s\ by\ s-a\ \ Second\ \ shifting\ rule$

$$\mathcal{L}\,(f(t-a)H(t-a)=e^{-as}\ \ \mathcal{L}\left(f(t)\right)\ ,$$ Second shifting rule

$\mathcal{L}\,(g(t)H(t-a) = e^{-as}\ \mathcal{L}\left(g(t+a)\right)$ *First and scond forms*

$$\mathcal{L}\left(f(t)\right) = \frac{\int_0^P \ \left(f(t)\right)e^{-st}\,dt}{1-e^{-Ps}} \quad Rule\ for$$

$P - periodic\ functions\ Assumed\ there\ Is\ f(t+P) = f(t)$

$$\mathcal{L}\left(f(t)\right)\mathcal{L}\ \left(g(t) = \mathcal{L}\,(f * g)(t)\right) \quad Convolution\ rule$$

$$Define\ (f\ .\ g)(t) = \int_0^t f(x)g(t-x)dx.$$

2.2 . *Some Examples of Laplace Transform Rules*

Example.2.2.1 *(Harmonic oscillator) Solve by Laplace's method the initial value problem x'' + x = 0 , x(0) = 0 , x'(0) = 1*

Solution*: The solution is x(t) = sin t. The details:*

ℒ (x'') + ℒ (x) = ℒ (0) *Apply ℒ across the equation*

S ℒ (x') – x'(0) + ℒ (x) =0

s[s ℒ (x) – x(0)] – x'(0) + ℒ (x) = 0 *Use again the t - derivative rule.*

$(s^2\ +\ 1)\mathcal{L}(x) =\ 1$ $Use\ x(0)\ =\ 0, x'(0)\ =\ 1$

ℒ (x) = $\frac{1}{s^2+1}$ *Divide.*

ℒ (sint) *Basic Laplace table*

.

$x(t) = sint$ $\qquad$ $Invoke\ Lerch's\ cancellation\ law.$

$\boldsymbol{Example2.2.2}$

$(s - differentiation\ rule)\ Show\ the\ steps\ for \mathcal{L}\ (t^2 e^{5t}) = \frac{2}{(s-5)^3}$

Solution:

$$\mathcal{L}(t^2 e^{5t}) = \left(-\frac{d}{ds}\right)\left(-\frac{d}{ds}\right)\mathcal{L}(e^{5t}) \qquad Basic\ Laplace\ table$$

$$= \frac{d}{ds}\left(\frac{-1}{(s-5)^2}\right) \qquad Calculus\ power\ rule.$$

$$= \frac{2}{(s+3)^3} \qquad Identity\ verified.$$

Example2.2.3

$(First\ shifting\ rule) Show\ the\ steps\ for \mathcal{L}(t^2 e^{-3t}) \ = \frac{2}{(s+3)^3}$

Solution:

$$\mathcal{L}\ (t^2 e^{-3t}) \ = \mathcal{L}(t^2)\lfloor s - s - (-3) \qquad First\ shifting\ rule.$$

$$= (\frac{2}{(s^2+1)})\lfloor s - s - (-3) \qquad Basic\ Laplace\ table.$$

$$= \frac{2}{(s+3)^3} \qquad Identity\ verified.$$

Example2.2.4. *(Second shifting rule) Show the steps for*

$$\mathcal{L}(\sin tH(t-\pi)) = \frac{e^{-\pi s}}{s^2+1} \dots \dots \dots (2.1)$$

$\boldsymbol{Solution}: The\ second\ shifting\ rule\ is\ applied\ as\ follows.$

$\mathcal{L}\,(\sin tH(t - \pi)) \ = \ \mathcal{L}\,(g(t)H(t - \ a)\ Choose$

$g(t) \ = \ sint, a \ = \ \pi \ = \ e^{-as}\mathcal{L}\,(g(t \ + \ a) \quad Second\ form$

$, second\ shifting\ theorem.$

$= e^{-\pi s}\mathcal{L}\,(\sin(t + a) \quad Substitute\ a = \ \pi. = \ \ e^{-\pi s}\,\mathcal{L}\,(-sin\ t)$

$Sum\ rule\ \ sin(a \ + \ b) \ = \ sin\ a\ cos\ b \ +$

$\sin b \cos a\ plus \sin \pi \ = \ 0, \cos \pi$

$= \ e^{-\pi s}\frac{-1}{s^2+1} \qquad Basic\ Laplace\ table. Identity\ verified.$

$Example2.2.5$

$(Trigonometric\ formulas) Show\ the\ steps\ used\ to\ obtain$

$these\ Laplace\ identities:$

$(a)\ \mathcal{L}(t\cos at) = \frac{s^2-a^2}{(s^2+a^2)^2} \qquad (b)\mathcal{L}\,(t\sin at)\frac{2sa}{(s^2+a^2)^2}$

$(c)\ \mathcal{L}(t^2\cos at) = \frac{2(s^3-3sa^2}{(s^2+a^2)^3} \qquad (d)\mathcal{L}(t^2\sin at) = \frac{6s^2a-a^3}{(s^2+a^2)^3}$

Solution: *The details for (a):*

$\mathcal{L}\,(t\cos at) = -(d/ds)\mathcal{L}(\cos at)$ *Use s − differentiation.*

$$= -\frac{d}{ds}\left(\frac{s}{s^2+a^2}\right)$$ *Basic Laplace table.*

$$= \frac{s^2-a^2}{(s^2+a^2)^2}$$ *Calculus quotient rule.*

The details for (c)

$\mathcal{L}\,(t^2\cos at) = -(d/ds)\mathcal{L}\,((-t)\cos at)$ *Use s − differentiation*

.

$$= \frac{d}{ds}\left(-\frac{s^2-a^2}{(s^2+a^2)^2}\right)$$ *Result of ds*

$$= \frac{2s^3-6sa^2}{(s^2+a^2)^3}$$ *Calculus quotient ruleThe similar details for (b) and (d) are left as exercises.*

Example.2.2.6

(Exponentials)Show the steps used to obtain these Laplace identities:

$$(a)\mathcal{L}\,(e^{at}\cos bt) = \frac{s-a}{(s-a)^2+b^2}$$

$$(b)\mathcal{L}(e^{at}\sin bt) = \frac{b}{(s-a)^2+b^2}$$

$$(c)\mathcal{L}(te^{at}\cos bt) = \frac{(s-a)^2 - b^2}{((s-a)^2 + b^2)^2}$$

$$(d)\mathcal{L}(e^{at}\sin bt) = \frac{2b(s-a)}{((s-a)^2 + b^2)^2}$$

Solution: $Details\ for\ (a)$:

$\mathcal{L}\,(e^{at}\,cos\,bt))$

$= \mathcal{L}\,(cos\,bt)\lfloor s - s - a$ $\quad$ $First\ shifting\ rule.$

$= (\frac{s}{s^2+b^2})\lfloor s - s - a$ $\quad$ $Basic\ Laplace\ table.$

$= \dfrac{s-a}{(s-a)^2 + b^2}$ $\quad$ $Verified$

Details for (c):

$\mathcal{L}\,(te^{at}\,cos\,bt) = \mathcal{L}\,(tcos\,bt)\lfloor s - s - a$ $\quad$ $First\ shifting\ rule.$

$= (-\dfrac{d}{ds}\mathcal{L}(\cos bt)\lfloor s - s - a$ $\quad$ $Apply\ s\ - differentiation.$

$= (-\dfrac{d}{ds}\left(\dfrac{s}{s^2+b^2}\right))\lfloor s - s - a$ $\quad$ $Basic\ Laplace\ table.$

$= \left(\dfrac{s^2 - b^2}{(s^2+b^2)^2}\right)\lfloor s - s - a$ $\quad$ $Calculus\ quotient\ rule.$

$= \dfrac{(s-a)^2 - b^2}{((s-a)^2 + b^2)^2}$ $\quad$ $Verified$

Example.2.2.7. *(Hyperbolic functions) Establish these Laplace transform facts about* $\cosh u = (e^u + e^{-u})/2$ *and* $\sinh u = (e^u - e^{-u})/2$.

(a) $\mathcal{L}(\cosh at) = \frac{s}{s^2-a^2}$ (c) $\mathcal{L}(t\cosh at) = \frac{s^2+a^2}{(s^2-a^2)^2}$

(b) $\mathcal{L}(\sinh at) = \frac{a}{s^2-a^2}$ (d) $\mathcal{L}(t\sinh at) = \frac{2as}{(s^2-a^2)^2}$

Solution: *The details for (a):*

$$\mathcal{L}(\cosh at) = \frac{1}{2}\left(\mathcal{L}(e^{at}) + \mathcal{L}(e^{-at})\right)$$

Definition plus linearity of $\mathcal{L}$.

$$= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right)$$

Basic Laplace table.

$$= \frac{s}{s^2 - a^2}$$

Identity (a) verified.

The details for (d):

$$\mathcal{L}(\sinh at) = -\frac{d}{ds}\left(\frac{a}{s^2-a^2}\right)$$

Apply the s -differentiation rule.

$$= \frac{a(2s)}{(s^2 - a^2)^2}$$

Calculus power rule; (d) verified.

Example2.2.8

$(s-differentiation)Solve\ \mathcal{L}\big(f\ (t)\big)=\dfrac{2s}{(s^2+1)^2}\ \ for\ f(t).$

Solution: $The\ solution\ is\ f(t)=\ tsin\ t. The\ details:$

$$\mathcal{L}\big(f\ (t)\big)=\frac{2s}{(s^2+1)^2}$$

$=-\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right)$ $\quad Calculus\ power\ rule\ (u^n)'\ =\ nu^n-lu'.$

$=-\dfrac{d}{ds}(\ \mathcal{L}(tsin\ t)$ $\quad Basic\ Laplace\ table.$

$=\mathcal{L}(tsin\ t)$ $\quad Apply\ the\ s\ -differentiation\ rule.$

$f(t)\ =\ tsint$ $\quad Lerch's\ cancellation\ law.$

Example2.2.9.

$(First\ shift\ rule)\ Solve\ \mathcal{L}(f(t))\ =\ \dfrac{s+2}{2^2+2s+2}, for\ f(t).$

Solution:

$The\ answer\ is\ f(t)\ =\ e^{-t}cost\ +\ \ e^{-t}sint. The\ details:$

$\mathcal{L}\big(f\ (t)\big)=\dfrac{s+2}{2^2+2s+2}$ $\quad Signal\ for\ this\ method:$

$the\ denom-\ inator\ has\ complex\ roots.$

$=\dfrac{s+2}{(s+1)^2+1}$ $\quad Complete\ the\ square, denominator.$

$= \frac{s+1}{s^2+1}$ *Substitute S for s + 1.*

$= \frac{s}{s^2+1} + \frac{1}{s^2+1}$ *Split into Laplace table entries.*

$= \mathcal{L}(\cos t) + \mathcal{L}(\sin t)\lfloor s \to S = s+1$ *Basic Laplace table.*

$== \mathcal{L}(e^{-t}\cos t) + \mathcal{L}(e^{-t}\sin t)$ *First shift rule.*

$f(t) = e^{-t}\cos t + e^{-t}\sin t$

Invoke Lerch's cancellation law.

Example.2.2.10

(Damped oscillator) Solve by Laplace's method the initial value problem x" + 2x' + 2x = 0,x(0) = 1, x'(0) = -1

Solution: *The solution is* $x(t) = e^{-t}\cos t$. *The details:*

$\mathcal{L}(x") + 2\mathcal{L}(x') + 2\mathcal{L}(x) = \mathcal{L}(0)$

Apply $\mathcal{L}$ *across the equation.*

$S\mathcal{L}(x') - x'(0) + 2\mathcal{L}(x') + 2\mathcal{L}(x) = 0$

The t – derivative rule on x'.

$s[S\mathcal{L}(x) - x(0)] - x'(0)$ *The t – derivative rule on*

$x + 2[\mathcal{L}(x) - x(0)] + 2\mathcal{L}(x) = 0$

$(s^2 + 2s + 2)\mathcal{L}(x) = 1 + s$ *Use* $x(0) = 1, x'(0) = -1$

$\mathcal{L}(x) = \frac{s+1}{s^2+2s+2}$ *Divide.*

$$= \frac{s+1}{(s+1)^2+1} \qquad \textit{Complete the square in the de nominator.}$$

$$= \mathcal{L}(cost)\lfloor s - s + 1 \qquad \textit{Basic Laplace table.}$$

$$= \mathcal{L}(e^{-t}cost) \qquad \textit{First shifting rule.}$$

$$x(t) = e^{-t}cost \qquad \textit{Invoke Lerch's cancellation law.}$$

Example2.2.11

(Rectified sine wave) Compute the Laplace transform of the rectified sine wave $f(t) = |\sin wt|$ *and show it can be expressed in the form*

$$\mathcal{L}(|\sin wt|) = \frac{w \coth(\frac{\pi s}{2w})}{s^2 + w^2}$$

Solution:

The periodic function formula will be applied with period $P = 2\pi/w$. *The calculation reduces to the evaluation of* $J = \int_0^p f(t)e^{-st}\, dt$. *Because* $\sin wt \leq 0$ *on* $\pi/W \leq t \leq 2\pi/w$, *integral J can be written as* $) = J_1 + J_2$, *where*

$$J_1 = \int_0^{\pi/w} \sin wte^{-st}\, dt, J_1 = \int_{\pi/w}^{2\pi/w} - \sin wte^{-st}\, dt$$

Integral tables give the result

$$\int \sin wte^{-st} = -\frac{we^{-st}\cos(wt)}{s^2+w^2} - \frac{we^{-st}\sin(wt)}{s^2+w^2}$$

Then

$$J_1 = \frac{w(e^{-\pi*s/w} + 1)}{s^2 + w^2}, J_2 = \frac{w(e^{-2\pi*s/w} + e^{-\pi*s/w})}{s^2 + w^2}$$

$$J = \frac{w(e^{-\pi*s/w} + 1)^2}{s^2 + w^2}$$

The remaining challenge is to write the answer for

$\mathcal{L}\big(f\ (t)\big)$ *in terms of coth. The details:* $\mathcal{L}(f\ (t))$

$$= \frac{J}{1 - e^{-ps}} \quad Periodic\ function\ formula.$$

$$= \frac{J}{(1 - e^{-\frac{ps}{2}})(1 + e^{-\frac{ps}{2}})} \quad Apply\ 1 - x^2 = (1 - x)(1 + x),$$

$$= \frac{w(1+e^{-\frac{ps}{2}})}{(1-e^{-\frac{ps}{2}})(s^2+w^2)} Cancel\ factor\ 1 + e^{-Ps/2}$$

$$= \frac{e^{-\frac{ps}{4}}+e^{-\frac{ps}{4}}}{e^{-\frac{ps}{4}}-e^{-\frac{ps}{4}}} \frac{w}{s^2+w^2} \quad Factor\ out\ e^{-Ps/4}, then\ cance$$

$$= \frac{2\cosh(\frac{ps}{2})}{2\sinh(ps/4)} \frac{w}{s^2+w^2} \qquad Apply\ cosh, sinh\ identities.$$

$$= \frac{w\ cosh\left(\frac{ps}{2}\right)}{s^2+w^2} Use\ \coth u = \frac{coshu}{\sinh u}.$$

$$= \frac{w\ cosh\left(\frac{\pi s}{2w}\right)}{s^2 + w^2} Identity\ verified.$$

Example2.2.12

$(Half - wave\ rectification) Compute\ the\ Laplace$

$transform\ of\ the\ half - wave\ rectification\ of\ \sin wt,$

$denoted\ g(t), in\ which\ the\ negative\ cycles\ of\ \sin wt\ have$

$been\ canceled\ to\ create\ g(t). Show\ in\ particular\ that$

$$\mathcal{L}\big(g(t)\big) = \frac{1}{2}\frac{w}{s^2 + w^2}(1 + \coth(\frac{\pi s}{2w}))$$

Solution:

$The\ half - wave\ rectification\ of\ sin\ wt\ is\ g(t)\ =$

$(sin\ wt\ +\ |sin\ wt|)/2.$

$Therefore, the\ basic\ Laplace\ table\ plus\ the\ result\ of\ Example$

$21\ give$

$$\mathcal{L}(2g(t))\ =\ \mathcal{L}(sin\ wt)\ + \mathcal{L}(\ sin\ wt|)$$

$$= \frac{w}{s^2 + w^2} + \frac{w \cosh(\frac{\pi s}{2w}))}{s^2 + w^2}$$

$$= \frac{w}{s^2 + w^2}(1 + coth(\frac{\pi s}{2w}))$$

$Dividing\ by\ 2\ produces\ the\ identity.$

Example2.2.13

$(Shifting\ rules)\ Solve\ \mathcal{L}(f(t))\ =\ e^{-3s}\frac{s+1}{s^2+2s+2}\ for\ f(t).$

Solution:

$The\ answer\ is\ f(t)\ =\ e^{3-t}cos(t-3)H(t-3).The\ details:$

$$\mathcal{L}\big(f(t)\big) = e^{-3s}\frac{S+1}{(S+1)^2+1} \qquad Complete\ the\ square.$$

$$= e^{-3s}\frac{S}{s^2+1} \qquad Replace\ s\ +\ 1\ by\ S.$$

$$= e^{-3s+3}(\mathcal{L}(\cos t)\lfloor s \to s = s+1 \qquad Basic\ Laplace\ table.$$

$$= e^{3}(e^{-3s}(\mathcal{L}(cos\ t))\lfloor s \to s = s+1\ \ Regroup\ factor\ e^{-3s}$$

$$= e^{3}\Big(\mathcal{L}\big(cos(t-3)H(t-3)\big)\Big)\lfloor s - S = s+1$$

$$second\ shifting\ rule.$$

$$= e^{3}\mathcal{L}\big(e^{-t}\cos(t-3)\,H(t-3)\big) \qquad First\ shifting\ rule.$$

$$f(t) = e^{\,3-t}cos(t-3)H(t-3) \quad Lerch's\ cancellation\ law$$

.

Example2.2.14 $Solve\ \mathcal{L}(f(t)\ = \frac{s+7}{s^2+4s+8}\quad for\ f(t).$

Solution:

$$The\ answer\ is\ f(t) =\ e^{-2t}\ (\cos 2t + \frac{5}{2}\sin 2t)\ .$$

$The\ details: \mathcal{L}\ (f(t)\ = \dfrac{s+7}{(s+2)^2+4}\quad Complete\ the\ square.$

$= \frac{s+5}{s^2+4}$ $\qquad Replace\ s\ +\ 2\ by\ S.$

$= \frac{s}{s^2+4} + \frac{5}{2}\frac{2}{s^2+4}$ $\qquad Split\ into\ table\ entries.$

$$= \frac{s}{s^2+4} + \frac{5}{2}\frac{2}{s^2+4}\lfloor s \to s = s+2$$

$Prepare\ for\ shifting\ rule.$

$$=\ \mathcal{L}(\cos\ 2t) + \frac{5}{2}\mathcal{L}(\ sin2t)\lfloor s \to s = s+2$$

$Basic\ Laplace\ table.$

$=\ \mathcal{L}(e^{-2t}(cos\ \ 2t + \frac{5}{2}sin2t$ $\qquad First\ shifting\ rule.$

$f(t) = e^{-2t}(cos\ \ 2t + \dfrac{5}{2}sin2t\ )$ $\quad Lerch's\ cancellation\ law.$

Conclusion

The use of Laplace transforms to solve some serious value problems in differential equations is an important topic that is no less important than the rest of the topics related to mathematics as it specializes in the problems that have been solved by the Laplace transform. Laplace came up with the .possibility of solving boundary value problems in one step

This does not mean that using normal methods is not useful, but in order to reach the solution in an easier way

And faster, in addition to the stressful fatty processes, and as you know perfectly well that mathematics is a cumulative science that depends on

What was previously reached, without reaching the normal method, we would not have reached other methods such as

Laplace transform and other methods, leave the topic of research development and come up with easier methods and shorten it

Laplace's field for future generations, God willing

researcher

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